

B.Sc. Semester - 6 (CBCS) Examination

March/April-2025 (NEW COURSE)

Graph Theory and Complex Analysis-2 (Core (New))

Time: 2:30 Hours

Marks: 70

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1(A) ATTEMPT ANY TWO:

(10)

- (1) Write a short note on *Königsberg Bridge* problem.
- (2) Prove that if a graph has exactly two vertices of odd degrees, there must be a path joining these two vertices.
- (3) Prove that a graph $G = (V, E)$ is disconnected *iff* its vertex set V can be partitioned into two nonempty, disjoint subsets V_1 & V_2 such that there exists no edge in G whose one end vertex is in subset V_1 and the other in subset V_2 .

Que-1(B) ATTEMPT ANY ONE :

(04)

- (1) Define: (i) Subgraph (ii) Vertex-disjoint Subgraphs (iii) Closed Walk (iv) Euler graph.
- (2) A simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

Que-2(A) ATTEMPT ANY TWO:

(10)

- (1) Define: Planar Graph
Prove that Kuratowski's second graph is non-planar.
- (2) State and prove the Euler's theorem for planar graphs.
- (3) Explain Decyclization by taking suitable directed graph using Boolean Arithmetic.

Que-2(B) ATTEMPT ANY ONE :

(04)

- (1) If G be a graph with n vertices, e edges and f faces, then prove that
(i) $e \geq \frac{3}{2}f$ (ii) $e \leq 3n - 6$.
- (2) Define: Incidence Matrix.
Prove that the rank of the incidence matrix $A(G)$ of a connected graph G with n vertices is $n - 1$.

Que-3(A) ATTEMPT ANY TWO:

(10)

- (1) Discuss the mapping $w = e^z$ in polar and Cartesian forms.
- (2) Show that every linear fractional transformation except the identity transformation $w = z$, has at most two fixed points in the extended plane.
- (3) Show that Bilinear mapping $w = \frac{az+b}{cz+d}$, $z \neq -\frac{d}{c}$, a, b, c & $d \in \mathbb{C}$,
 $ad - bc \neq 0$ is conformal.

Que-3(B) ATTEMPT ANY ONE : (04)

- (1) Find the images of the lines $y = x - 1$ and $y = 0$ under the transformation $w = \frac{1}{z}$.
- (2) Prove that the set of all Mobius transformations forms a group with respect to the binary operation of composition of mappings.

Que-4(A) ATTEMPT ANY TWO: (10)

- (1) Define: Region & Radius of Convergence of Series.
Find region and radius of convergence for $\sum_{n=1}^{\infty} \frac{z^n}{2^{n+1}}$.
- (2) Define: Analytic Function
State and prove Taylor's infinite series of an analytic function $f(z)$.
- (3) Expand $\frac{2z^3+1}{z^2+z}$ in Taylor's Series at $z_0 = i$.

Que-4(B) ATTEMPT ANY ONE : (04)

- (1) Show that $\frac{1}{z^2} = \frac{1}{4} + \frac{1}{4} \sum_{n=1}^{\infty} (-1)^n (n+1) \left(\frac{z-2}{2}\right)^n$.
- (2) Expand $\frac{1}{z(1-z^2)}$ in Laurentz's series for $|z| < 1$.

Que-5(A) ATTEMPT ANY TWO: (10)

- (1) If z_0 is the m^{th} ($m \geq 2$) order pole of a complex function $f(z)$, then prove that $Res(f(z), z_0) = \frac{\varphi^{m-1}(z_0)}{(m-1)!}$, where $\varphi(z) = (z - z_0)^m f(z)$ is an analytic function and $\varphi(z_0) \neq 0$.
- (2) State and prove the Cauchy's residue theorem.
- (3) Evaluate: $\int_C \frac{3z^3+2}{(z-1)(z^2+9)} dz$, where $C: |z-2| = 2$.

Que-5(B) ATTEMPT ANY ONE : (04)

- (1) Evaluate: $\int_0^{\infty} \frac{1}{(x^2+1)^2} dx$.
- (2) Evaluate: $\int_0^{\pi} \frac{d\theta}{(2+\cos\theta)^2}$.
